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Temporal Asymmetry as Precondition of Experience. The Foundation of the Arrow of Time

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Knowledge as knowledge of facts is based on the appearance of time as fundamental difference between past and future. Taking asymmetry in time as precondition of experience, the time symmetry of the basic laws of physics has to be explained. We give an explanation founded on the abstraction leading from semigroups to groups and the essential role of the concept of groups to define stationary states and observables. We then present a short outline of the construction of an abstract quantum theory as theory of knowledge based on the asymmetry between facts and possibilities. It is well known that the second law of thermodynamics cannot be derived from the H-theorem without a further hypothesis. We show that the application of the concept of probability to the past yields inconsistency of the H-theorem and a derivation of the second law via a Boltzmann-type hypothesis. The question remains whether the distinction of facts and possibilities as precondition of a theory of knowledge is rooted in a theory of cognition itself.

1. INTRODUCTION

Two persistent attractors of discussion of fundamental problems in physics are the temporal asymmetry, often called the "arrow of time," and a satisfactory interpretation of the act of measurement in quantum theory. Usually the questions concerning these topics are put in the following ways:

(a) The fundamental equations of physics are time symmetric, i.e., they are invariant under time inversion (we exclude K^0 decay here). What causes the empirically observed temporal asymmetry? Is it based on extratheoretical initial conditions which may be of cosmological origin?

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(b) There are two time evolutions in quantum theory: a continuous one governed by the Schrödinger equation and a discontinuous one, the so-called collapse of the wave function. What causes the collapse? The measuring apparatus, the observer, or even the consciousness of the observer?

The idea we pursue here is that the two topics are related and that their relation can be revealed by focusing on a different attractor: the understanding of the process we call (human or scientific) knowledge. Undoubtedly, quantum theory is science, hence part of knowledge. But we try to go beyond this with the claim that quantum theory is also a theory of human (or at least scientific) knowledge itself.

Our starting point is that knowledge as knowledge of facts is based on the structure of time as fundamental difference between past and future. This assumption is *not*—as it may seem at a first inaccurate view—the avoidance of the questions posted above by changing them into suppositions. Instead, it is a *reversal* of the questions above.

We formulate this as follows.

Thesis 1. In quantum theory considered as theory of knowledge itself, asymmetry in time is central and time symmetry has to be explained.

Formally, the fundamental equations of classical mechanics are time reversible because they are second-order differential equations; the Schrödinger equation is a first-order equation in the time variable, but for a complex-valued function. This points to a surprising link between the following two questions:

A. Why are the fundamental laws of physics time-symmetric?
B. Why, though all our measurements give real-valued results, is quantum theory defined in a complex vector space?

The "why" in both questions originates from a way of thinking which should not only like to accept our theories developed in history, but wants to base them on simple and evident postulates. This, of course, refers to the difficult inquiry of what we actually adopt as "explanation" of a theory, i.e., to which postulates we ascribe the most explicative power.

Our assumption is that the structures appearing in A and B rest on the fundamental role of the mathematical concept of groups in the transition from the natural to the complex numbers. In Section 2 we incorporate this assumption in a second thesis which we carry out in three steps. First, we discuss the role of the extension of a semigroup to a group in number theory. Second, we display our thesis in connection with classical physics and historically developed quantum theory. Third, in Section 3, we give a short outline of a reconstruction of quantum theory from simple postulates, where the mathematical concepts in question play an essential part and the view of quantum theory as theory of knowledge becomes evident. In Section 4 we come back to our central thesis of time asymmetry and consider some approaches where the distinction between past and future is introduced explicitly as logically independent of the time-symmetric theories. Section 5 contains concluding remarks.

2. NUMBER THEORY AND THE EVOLUTION OF PHYSICS

We start with the following statement.

Thesis 2. The development from natural to complex numbers is based on the reversal of algebraic operations which define semigroups. The resulting group structures and algebraic completeness are needed in the mathematical formulation of physics.

Next we give a short outline of the construction of the concept of numbers:

(a) *Natural numbers* **N**. Natural numbers can be defined as classes of equivalent classes. Thereby we get a natural order: first the zero set as set of all objects with self-contradicting properties; then—from n to $n+1$—the class of all sets which are equivalent to the set of numbers defined up to n.

(b) *Whole numbers* **Z**. The operation of addition (+) of natural numbers defines a commutative semigroup (**N**, +) which—considered as ordered set—is isomorphic to the set of natural numbers. This semigroup can be enlarged to a group via the inverse operation "subtraction." The elements of this group are the whole numbers. We want to stress the point that we do not consider here the whole numbers as an extension of the natural numbers, but as a group operating on the natural numbers.

Next we introduce the algebraic operation called multiplication.

(c) *Rational numbers* **Q**. The operation of multiplication acting on the semigroup $(\mathbf{Z}\backslash\{0\}, +)$ defines a semigroup $(\mathbf{Z}\backslash\{0\}, \bullet)$ which can be enlarged to a group via the inverse operation "division." The elements of this group are the positive rational numbers **Q**.

(d) *Algebraic complex numbers* **A**. The operation of potentiation (with fixed basis) acting on the group **Q** defines again an automorphism semigroup which can be enlarged to a group via the inverse operation "radification." The elements of this group form the algebraic complex numbers, which are algebraically complete, i.e., the fundamental theorem of algebra is valid.

Usually the rational and the algebraic complex numbers are extended into the real and the complex numbers, respectively, by adding all limit points of all convergent Cauchy sequences.

The application and the structuring effect of mathematics onto physics has its roots in the existence of objects, predicates, and events in the empirical world which can be characterized (to good approximation) by mathematical concepts, especially numbers. In the following we demonstrate briefly the historical development of physics in a way which shows close connections with the development of number theory.

(a′) The concept of a class or a set presupposes that the collected elements are individually recognizable and distinguishable. The application of the concept of numbers to experience rests on the tacit assumption that this discrimination of elements is a property of reality. This yields the possibility to count objects or events. The continuation of counting, i.e., the semigroup character of addition, shows a natural relation to time in its modalities. One has counted up to n; this is a present fact based on past action; one may count further; this is a possibility pointing into future. Kant and later Brouwer (1908) explained the evidence of arithmetics as founded in the pure intuition of time, *a priori*, independent of the special content of experience. Counting of distinguishable, temporal successive events can—in principle—be continued without limits.

(b′) It is our experience that the sun rises every day. But this experience is—considered rigorously—experience of past events, of facts. It was Hume's merit to show that no *logical* conclusion can be drawn from past to future sunrises. But in practice, we expect daily sunrises in future. And as far as human knowledge extends, there were always sunrises. Sunrises are an ordered class of events which seem to be unlimited in past and future; therefore they can comfortably be marked by whole numbers; the semigroup structure is replaced by the group. Here, we have the empirical basis of reversibility. Unlimited prediction is supplemented by pragmatically unlimited retrodiction.

(c′) Extended objects are divisible. As long as no undivisible smallest parts (atoms) are known, it suggests itself to characterize the parts by rational numbers. Already Aristotle defines the continuum as that which can be partitioned in an unlimited way. The abstract structure of unlimited divisibility is described as "space." Its geometrical analysis leads to the introduction of real numbers. Describing also time as measured by clocks as a real-valued continuum, physical laws can be formulated by differential equations. The symmetry of these equations is then studied by group theory. Symmetry operations such as reversal of motion, for example, leave the equations invariant and map solutions onto (in general different) solutions. Irreversible processes appear as special solutions, the empirically known irreversibility as distinguishing special types of solution. Having this in mind, it seems obvious to consider reversibility as fundamental and the distinction of special types of solutions as something which needs to be explained.

(d′) Question B, why quantum theory is defined in a complex vector space, was already the subject of a dialogue between Ehrenfest (1932) and Pauli (1933). We suppose [as described in detail in Drieschner *et al.* (1987)] that the use of a complex vector space is a consequence of the existence of a symmetry transformation group which leaves a certain well-defined probability metric invariant (see also Section 3). Group representations based on linear representations in complex vector spaces have—because of the fundamental theorem of algebra—the advantage that the representation operators are diagonalizable. This allows the distinction of a basis of one-dimensional invariant subspaces as *stationary* states with respect to the symmetry transformations. The generators of these transformations can be considered as "observables," i.e., as mathematical representatives of observable quantities, their eigenvalues being possible outcomes of measurements. In this view, quantum theory turns out to be a *general* theory connecting measurement values and quantities which can be predicted with probability.

3. RECONSTRUCTION OF QUANTUM THEORY

The parallelism in the development of physical theories and number theory may seem surprising. Why do such classes of objects, predicates, and events exist in reality which represent groups derived via inverse operations from semigroups approximately? Our methodological assumption is that from a reconstruction of physics, i.e., a subsequent systematic construction, the applicability of these groups can be deduced from simple postulates. For a detailed treatment of reconstruction see Drieschner *et al.* (1987). Here we present only a short outline in view of the preceding considerations. We start with no presupposition of a set of laws of "classical" physics which would then have to be "quantized." Our central point is the reversal of the usual argumentation. First, preconditions of experience are formulated and abstruct quantum theory as a theory of probabilistic predictions on empirically decidable alternatives is reconstructed. The concepts of position space and objects (particles) are derived as consequences.

Reconstruction of abstract quantum theory is based on three postulates, which we present shortly. To do this, we have to define some concepts first.

Facticity of the past: Past events are considered as objective facts, independently of our actual knowledge of them.

Possibility of the future: We are aware of future events only as possibilities.

Temporal statements: A temporal statement is a verbal proposition (or a mathematical proposition with physical meaning) referring to a moment in time.

These definitions express the modalities of time which we consider as a precondition of experience.

Probability: Probability is a quantification of possibility, defined as prediction (expectation value) of a relative frequency.

State: A state is a recognizable event. A state is what is the case when some temporal statement is true.

Conditional probability: Let x and y be two states. Then $p(x, y)$ is the probability to find y and x is present.

Alternative: An n-fold alternative is a set of n mutually exclusive states, exactly one of which will turn out to be present if and when an empirical test of this alternative is made. In good approximation we can divide reality into different parts (objects) and only parts of reality (not the whole of it) can be caught by conceptual thinking. Therefore we have the following first postulate:

Postulate 1. There are empirically decidable finite alternatives.

This is itself an approximation, but an approximation which is a precondition of every empirical decision. It is the first example of an abstraction which causes the applicability of mathematical concepts in physics [cf. (a) and (a′), Section 2].

As second postulate we have the following:

Postulate 2. The states of alternatives change continuously in time. Thereby the conditional probabilities are not altered.

This is also called the "Darwinism of states." Hypothetical states not fulfilling this postulate are hardly reproducible in observation.

To correct and connect the two preceding postulates, we need a third one. This has to bridge the discontinuity of the discrete alternatives and the continuous time development. We call this the postulate of indeterminism.

Postulate 3. To every alternative of two states x and y there exist states z which possess conditional probabilities $p(z, x)$ and $p(z, y)$ none of which are equal to zero or one.

As consequences of these postulates one can derive a space of mutually equivalent states. This is a representation space for a symmetry group which preserves the probabilities between the states and further on a probability metric on this space. A possible dynamics will be given by a one-parameter subgroup, i.e., this subgroup is the mathematical representation of time. To get observable states for this dynamics, the one-parameter subgroup has to be diagonalizable. Hence the state space must be a unitary one, i.e., a complex Hilbert space. For further details see Drieschner *et al.* (1987).

4. TIME-SYMMETRIC THEORIES AND THE POSTULATE OF TIME ASYMMETRY

In this section, we come back to our main thesis, which states that for a theory of knowledge, asymmetry in time is central. We conjecture that this fundamental appearance of time—the asymmetry between facts and possibilities—is the root of the conformity in direction of the so-called thermodynamic, cosmological, and psychological arrows of time. It should then be possible to deduce the directional identity. Steps in this direction have been done with respect to the cosmological arrow in Görnitz (1988) and Görnitz and Ruhnau (1989) and with respect to the psychological arrow in Ruhnau and Pöppel (1991). For more details, we refer to these publications. In the following we discuss some approaches where the distinction between past and future is explicitly formulated and introduced as *logically independent* of the time-symmetric theories.

First we discuss Boltzmann's H-theorem and the second law of thermodynamics. We show that the statistical interpretation of this law is exactly the point where the structure of time as distinction between facts and possibilities manifests itself. These considerations are based on Weizsäcker (1939).

As is well known, the thermodynamic arrow of time is not a direct consequence of Boltzmann's H-theorem. To get the second law of thermodynamics one needs an extra condition, usually an initially low entropy which might be of cosmological origin. The H-theorem itself does not distinguish a direction in time, it only states that given an isolated system whose entropy is not maximal at a certain moment of time t_0, the probability that the entropy at any time $t \neq t_0$ is greater than the entropy at t_0 is immense. Therefore with overwhelming probability, one can deduce that the entropy of the system is greater for $t > t_0$ than for t_0. But with the same overwhelming probability one can show that the entropy was greater for $t < t_0$. This contradicts the second law, which requires a smaller or at least equal entropy value for the past.

Now our considerations here are based on the fundamental distinction between facts and possibilities, the quantification of possibilities being probabilities. Past events as facts are not characterized by probabilities. Ascribing probabilities to past events is incomplete knowledge. This leads us to the following conclusion, which corresponds to an early conjecture of Gibbs (1905).

Conclusion. The second law of thermodynamics is consistent with the H-theorem only in the case of future entropy values of present known systems are derived from the H-theorem, never past entropy values. With this restriction, the second law is an immediate consequence of the H-theorem.

The conclusion is defended by the following reflection:

As demonstrated above, the second law of thermodynamics cannot be derived from the H-theorem. Let us assume a Boltzmann-type hypothesis, namely that long ago the state of our known universe was statistically highly improbable. Then—to the approximate to which the known universe can be considered an isolated system—the immediate conclusion is an entropy increase for all times subsequent to this initial state.

Now the enormous improbability of the initial state of low entropy causes the question of why on the one hand such an improbable state can be realized where on the other hand the statistical foundation of thermodynamics rests on the assumption that with a high degree of certainty only the probable states occur? This is a strange question because we are talking about the probability of an initial state which is—as far as we know—unique. But to prove this question as inadequate, one has to characterize the past in a way which does not contain the concept of probability as a fundamental concept. This is our claim expressed in Thesis 1.

Conversely, if one wants to keep the fundamental meaning which the concept of probability has in Boltzmann's formulation, one has to show that a proper statistics predicts the occurrence of this initial state. Following the argumentation which considers the known universe as a fluctuation (from thermal equilibrium) in a temporally and spatially more extended universe, we consider a state x_1 of our known universe at a time shortly after the state of smallest entropy x_0. From the H-theorem it follows that the probability of x_1 is greater than that of x_0. But then there should exist a much bigger quantity of single universes whose initial values are states x_1 than universes starting with x_0.

Therefore in the frame of a Boltzmann-type derivation of the second law, the application of the probability concept to the past leads to absurd consequences. This confirms our conclusion and also our main Thesis 1.

Taking the facticity of the past and the possibility of the future as fundamental, the second law is an immediate consequence. The H-theorem predicts an increase of entropy in future. But every past moment was once present, i.e., the increase of entropy can be deduced for all—at that time—future moments, especially for times which have passed now.

Facticity of the past and possibility of the future as precondition of experience is of interest also in the treatment of the measurement problem in quantum theory. We do not want to raise here the difficult question of the measurement process itself; for a detailed discussion see Weizsäcker and Görnitz (1990). Instead, we focus on the point that quantum theory yields statements about the probabilistic connections between successive observations. We refer to a paper by Aharonov *et al.* (1964) [see also Aharonov and Albert (1980), Albert *et al.* (1985), and Bub and Brown (1986) for further development and critique]. They argue that the time asymmetry implicit in the quantum theory of measurement is related to the way in which statistical ensembles are constructed. To show this, they first construct an ensemble time symmetrically by using both initial and final states of the system to delimit the sample. The calculated probability distribution is then time-symmetric as well. The expression for prediction can formally be recovered by preceding the final selection in the time-symmetric theory by "coherence-destroying" manipulations. Conversely, carrying out such procedures following the initial selection of a time-symmetric ensemble, they obtain a "retrodiction" formula. To recover the conventional prediction (which does not involve any postselection) from the time-symmetric formula, they adopt a postulate that is logically independent of the time-symmetric theory: "In our universe ensembles chosen on the basis of an initial complete measurement alone possess unambiguous and reproducible probability characteristics."

The time reverse of this postulate—being logically conceivable—has to be excluded for our universe.

At the end of this section we only mention two fairly new approaches. One is by Haag (1990), who introduces the following proposition: "The transmutation from possibilities into facts should be introduced as an essential element in fundamental theory."

The suggested theory is up to now very tentative; therefore we do not discuss it in detail.

Another proposal to consider time asymmetry as fundamental is Penrose's *Weyl curvature hypothesis*: A certain constraint—the vanishing of the Weyl curvature tensor—applies only to initial (past) space-time singularities, not to final (future) singularities. For final singularities one expects divergences of the Weyl tensor (Penrose, 1979).

With respect to all four approaches, ours is the most abstract.

5. CONCLUDING REMARKS

In the last section attention was paid to a clear distinction between the time symmetry of physical laws and the experienced asymmetry in time. It seems that symmetry and asymmetry in time are *logically* independent, that there is no logical derivation possible which proves symmetry from asymmetry and vice versa.

Taking time symmetry of fundamental physical laws as the basic aspect, one needs boundary conditions to attain asymmetry. But we should be aware of the fact that reversibility of the fundamental equations does not imply that we actually observe the time reversal of the evolution of a state starting with certain initial conditions; it only implies inversion of motion.

But on the other hand, any experience which is expressed in a conceptual way presupposes and rests on the concept named "fact." For our point of view in which quantum theory is considered as a theory of knowledge itself, i.e., not only as a conceptualization of experience, but also as a tool to make predictions, the distinction between facts and possibilities as precondition of experience is central. Based on this fundamental asymmetry in time, we gave an argument (not a logical derivation) which leads to time symmetry. The question remains whether the essential role of the group concept, whose applicability is deeply connected with the concepts of decidable alternatives and probabilities, and the necessity to define stationary states may be related to the structure of our cognition itself. If so, this opens far-reaching questions which are not only of philosophical interest, but which may also lead to new concepts in physics.

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