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The structures of interactions – How to explain the gauge groups U(1), SU(2) and SU(3)

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Abstract

It is very useful to distinguish between four types of interactions in nature: gravitation, and then electromagnetism, weak interaction and strong interaction. The mathematical structure of electromagnetism but also of weak and strong interaction could be understood as induced by a local gauge group. The associated groups are the unitary group in one dimension – U(1) – for electromagnetism, the special unitary group in two dimensions – SU(2) – for the weak interaction, and the special unitary group in three dimensions – SU(3) – for the strong interaction. The essence of this article is to give a "first-principles" explanation for the three gauge groups.

1 Introduction

Countless experiments have shown that it is very useful to distinguish between four types of interactions in nature.

The oldest one treated in physics is *gravitation*. Since Newton's days, it is clear that this force manages the interactions between all astrophysical objects. Its range is infinite. Later on, the theory was partially replaced by Einstein's theory of *general relativity*.

By the end of the 18th century, the physicists had learned more and more about electricity. At the beginning of the 19th century, it was evident that electricity and magnetism are connected. In the middle of the 19th century, Maxwell then clarified the mathematical structure of *electromagnetism*.

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In the middle of the 20th century, it became clear that it is possible to decompose the atomic nuclei into protons and neutrons. Therefore a force has to exist which is much stronger than the electromagnetic force and which is able to hold all the protons inside the nucleus – which repel one another by their electric charge.

On the other hand, the decay of the neutron and especially the discovery of the neutrino have given the hint for another interaction, the *weak interaction*.

Experimental results, at first from interactions of electrons with protons, showed that there are structures inside the proton. Today these structures are named quarks and gluons and are often described as "particles". The theory of *strong interaction* explicates now the behavior of quarks and gluons and the resulting forces between protons and neutrons.

Towards the end of the 20th century, it became clear that not only the mathematical structure of electromagnetism but also of weak and strong interaction could be understood as induced by a local gauge group. This means that the interaction shows symmetries, which are allowed to be differently oriented at different points in space.

The associated groups are the unitary group in one dimension – U(1) – for electromagnetism, the special unitary group in two dimensions – SU(2) – for the weak interaction, and the special unitary group in three dimensions – SU(3) – for the strong interaction.

With these three compact groups, the relevant structures that rule the related interactions can be described.

There are also efforts to apply the conception of a gauge group to gravitation. Various groups have been proposed which should solve the problem for the gravitational interaction. As a further complication, the use of non-compact groups is inevitable here. So far none of the envisaged Lie groups have afforded a successful solution, and the issue of the quantum theoretical description of gravitation is still seen as open. We have given arguments elsewhere, why in the end such attempts will not be successful. [2]

Experimental evidence shows that the respective gauge groups are well chosen, but there is still the need for a deeper foundation of their mathematical structures. To the best of our knowledge, a "first-principles" explanation for the three gauge groups could not be furnished so far. For instance, H. Lyre states[3]

> „Am Horizont sowohl der physikalischen als auch der philosophischen Untersuchungen über Eichtheorien deutet sich die noch völlig ungeklärte Frage nach einer noch tieferen Bedeutung der Konzeption der Eichtheorien an. Denn wenngleich das Eichprinzip – wie gezeigt – nicht zwingend auf nichtflache Konnektionen führt, so ist ja doch die in der kovarianten Ableitung vorgegebene Struktur des Wechselwirkungsterms auch für den empirisch bedeutsamen Fall nicht-verschwindender Feldstärken korrekt beschrieben. Diese Wechselwirkungsstruktur ist also tatsächlich aus der lokalen Eichsymmetrie-Forderung hergeleitet. Was aber ist der tiefere Grund für diese, zunächst rein formale Möglichkeit? Scheinbar handelt es sich um einen tiefliegenden und konzeptionell noch völlig unverstandenen Zusammenhang zwischen Raum und Wechselwirkung"
>
> (At the horizon of both the physical as well as philosophical studies on gauge theories the still completely open question emerges of the deeper meaning of the conception of gauge theories. While the gauge principle – as shown – does not necessarily lead to non-flat connections, the structure of the interaction term, as determined by the covariant derivation, is correctly described even in the empirically significant case of non-vanishing field strength. This structure of the interaction is, in fact, derived from the requirement of local gauge symmetry. But what is the deeper reason for this, so far only formal, possibility? Seemingly, there is a deeply rooted and conceptually completely unexplained relationship between space and interaction.)

Even years later the situation appeared not to be more satisfactory. Norbert Straumann stated[4]:

> „Das Standardmodell hat aber noch andere unbefriedigende Züge. Das beginnt schon damit, dass wir nicht verstehen, weshalb gerade seine und nicht andere Eichsymmetrien realisiert sind."
>
> (The standard model has even other unsatisfactory aspects. This starts with the fact that we do not understand why exactly its and no other gauge symmetries are realized.)

The purpose of present article is to give a "first-principles" explanation for the three gauge groups.

The first-principles explanation will be based on the simplest structure that is imaginable in quantum theory. Such structures are absolutely defined abstract quantum information (AQI) bits, which already for mathematical reasons are the simplest conceivable structures. The AQI bits are cosmological founded. This is the reason for their absolute character. They must be imagined as free of any special or definite meaning. Quantum bits as forms of information are normally seen as carrying meaning or connotations. To avoid such obvious misinterpretations, for the general discussion the notion "protyposis" (Greek: "pre-formation") has been introduced. The protyposis is an expression for a cosmologically founded absolute and abstract quantum

[2] Görnitz, Th. (2014)
[3] Lyre (2002)
[4] Straumann, N. (2005)

information, being free of meaning. The AQI bits of the protyposis are the most abstract structures conceivable. Sufficiently many of them may congregate to form material or energy quantum particles and ultimately also acquire meaning.

In a previous article[5] an explanation was given based on the protyposis concept for the two groups U(1) and SU(2) as the gauge groups of electromagnetic and weak interaction.

The different descriptions of interactions almost always use the term "particles". Even in cases where "fields" are addressed, the explanation is based on the insight of theoretical physics since Einstein's introduction of light quanta that the force fields can be understood as structures of quantum particles.

- "Particles" thus prove to be the fundamental structures when it comes to the discussion of interactions.

To the present day, the conceptions of particles have been dominating the thinking about the basic structures of reality. For example, the official website of CERN states:

> The model describes how everything that they observe in the universe is made from a few basic blocks called fundamental particles, governed by four forces.[6]

> The theories and discoveries of thousands of physicists since the 1930s have resulted in a remarkable insight into the fundamental structure of matter: everything in the universe is found to be made from a few basic building blocks called fundamental particles, governed by four fundamental forces. ...
> All matter around us is made of elementary particles, the building blocks of matter. These particles occur in two basic types called quarks and leptons. Each group consists of six particles, which are related in pairs, or "generations". The lightest and most stable particles make up the first generation, whereas the heavier and less stable particles belong to the second and third generations. ...
> The quantum theory used to describe the micro world, and the general theory of relativity used to describe the macro world, are difficult to fit into a single framework. [7]

A possible decay of the "basic blocks" does not seem to damage their alleged fundamentality. The fact that thinking is similar elsewhere, is demonstrated by DESY, (**D**eutsches **E**lektronen-**Sy**nchrotron – the German electron synchrotron) in Hamburg on their related website:

> What does the world consist of at the smallest level? What are the most fundamental particles of matter? Natural scientists have been looking into these basic questions since antiquity. In the course of their search, they have encountered ever smaller building blocks – first atoms, then atomic nuclei consisting of protons and neutrons, and finally tiny particles called quarks. Today, particle physicists are investigating the fundamental mysteries of the universe: what holds the cosmos together, and how do particles acquire their mass in the first place?[8]

Taking the conceptions seriously, then something 'fundamental' should not have any internal structure nor should it possibly be able to decay.

All this shows how vaguely the term "particle" commonly is defined.

2 The conceptions of a "particle" and a "quantum object"

For a particle there is a clear mathematical definition by Eugene Wigner:

- A particle is defined by the fact that its states span an irreducible representation of the Poincaré group.

From this mathematical structure it follows that a particle is distinguished by its mass – with values ranging between zero and infinite – and by a spin, which may have the values 0 , ½, 1, 3/2, 2, ... etc.

However, beyond mathematical physics, this definition can hardly be utilized. What does it mean in concrete terms?

The Poincaré group is the group of movements in the Minkowski space, which is the space in which the special theory of relativity operates. This means that time is comprised to form a unity with length, width and height so that the mutual change of these variables becomes visible in case of movements.

Furthermore, the mathematical conception of a particle implies that no internal structure can be observed, i.e., the object can be treated as a "point particle". As has been demonstrated previously[9], the mathematical structure of the state space of such a particle, being an irreducible representation oft the Poincaré group, can be constructed from the AQI bits of the protyposis. According to this construction, particles in the strict sense are

[5] Görnitz, Th. (2014)
[6] http://home.cern/about/physics Accessed 20 April 2016
[7] http://home.cern/about/physics/standard-model Accessed 20 April 2016
[8] http://www.desy.de/research/particle_physics/index_eng.html Accessed 20 April 2016
[9] Görnitz, Th., Graudenz, D., Weizsäcker, C. F. v. (1992), Görnitz, Th., Schomäcker, U. (2012), see also Appendix 1

defined as structureless objects that can move in the vacuum of the Minkowski space. If a quantum object can undergo decay, a possibly alleged "structureless-ness" would be questionable.

Taking all this into account, the light quanta, the photons, can be defined as massless particles. Neutrinos and electrons are generally understood as point particles, since no internal structure can experimentally be verified. Electrons are stable, and neutrinos do not decay, however, their three types can transform themselves into each other.

Although protons are stable, an internal structure has been known for a long time. Accordingly, the quarks were introduced to express this structure. It has to be noted that, on the one hand, the proton is no point particle, and, on the other hand, quarks cannot move in the vacuum.

Given this situation, a further clarification of the terms used here is indicated, to which the following distinction of quantum structures may contribute:

Quanta without rest mass always move in vacuum at the speed of light. The quanta known so far are the photons. If there are gravitons as quanta of gravitation, they will also be massless. This is backed by the fact that, in 2016, gravitation waves have been observed for the first time by a device on earth. Of course, linearized approximate solutions of Einstein's equations can be quantized analogously to the case of electrodynamics. However, so far all attempts to quantize the complete general theory of relativity have not shown any tangible success actually, and this speaks against the existence of gravitons.

Quanta with rest mass are able to stay in vacuum within a small spatial area. Their mass is recognizable by their inertia, i.e., by the effort needed to change their respective state.

Structure quanta represent a particularly interesting consequence of quantum theory. This family includes the phonons, the quanta of sound vibrations in solids, as well as the quarks already mentioned.

The phonon-electron interaction is one of the key elements in the ubiquitous modern electronics based on semiconductor physics. Sound in solids can be viewed as positive atomic nuclei swaying around their equilibrium positions, which influences the movement of the negatively charged electrons. Here, the phonons are the correct quantum-theoretical description of these oscillations; of course, they cannot appear in vacuum outside the solids.

In the scattering of electrons off protons, quarks act like point particles existing inside the protons. Trying to isolate a quark is similarly unsuccessful as trying to isolate a magnetic pole by breaking a magnet. In case of the magnet, a north pole and a south pole result at the breakpoint and thus two smaller magnets, but never an isolated pole. Similarly, the isolation of a quark can be compared to the task to cut the end piece of an elastic strap in such a way that only the end piece is obtained, without a minimal piece of the strap.

Obviously, structure quanta cannot appear as objects in the vacuum, and they should not be presented as such. In that sense, they are not particles. Nonetheless, within their respective context, they can act like real particles. Possible designations, other than "structure quanta", are "virtual particles" or "quasiparticles".

3 The types of interactions

Gravitation acts on everything that exists in the cosmos. Within the general theory of relativity, it is described as local change of the space-time with a local variation of the density.

The most important interaction for all chemical and biological processes is the *electromagnetic interaction*, which is produced by electric charge. It can be described as a local gauge theory with the U(1) group as gauge group.

These two interactions have, in principle, an infinite range and therefore can be perceived in everyday life. The other two interactions have very short ranges and can be experienced only at the level of microphysics.

The *strong interaction* is a gauge theory with the SU(3) group as gauge group; it provides for cohesion of the atomic nuclei.

The *weak interaction* with gauge group SU(2) induces, among others, the decay of neutrons and instable nuclei.

While one may try to pressure also gravitation into the structure of a local gauge interaction, the various groups to be used in these attempts cannot be compact like the gauge groups of the three other interaction types. However, despite decades-long efforts, no satisfying quantization of the general theory of relativity could be constructed. As already mentioned, arguments based on the fundamentals of quantum theory suggest that gravitation is to be understood as the local manifestations of a quantum cosmology. Accordingly, a quantization and, in particular, a gauge-theoretical formulation seem to be redundant for the general theory of relativity. A different situation applies, if only a linear approximation of this theory is considered, as in case of the gravitational waves, for which quantization is possible. [10]

As was discussed in a previous article[11], the emergence of the two gauge groups U(1), SU(2) and their mathematical structure can be deduced from the basic considerations regarding the fundamental structure of quantum theory, that is, the absolute, abstract and cosmologically defined quantum information, the AQI bits of the protyposis.

It is important to note that the AQI bits are devoid of any specific ‚meaning‘. In the discussion of Black Holes the „information paradox“ plays an important role[12]. After the transit of an object through the horizon of a Black Hole, at most the mass, angular momentum, and charge of that object can be recovered, whereas all other sorts of significance associated with the object are irretrievably lost. This loss of meaningful information is referred to as the information paradox. What is left out of consideration here is the fact that the objectizable part of the AQI bits constituting the object can still be determined. However, this refers to free-of-meaning quantum bits.

This discussion shows that the distinction between free-of-meaning quantum information, being an objective quantity, and meaning, always comprising subjective connotations, is essential; without such a distinction problems of comprehension will arise by necessity.

The information paradox is due to the mix-up of the two notions – which to avoid the different designations have been introduced. Objects falling into a black hole take the AQI bits of the protyposis – i.e. the quantum information they represent – with them into the interior. However, any ‘significance’ resulting from the context of their description gets lost, because any context is interrupted by the existence of the horizon. So, the abstract and free-of-meaning information of the AQI bits survives this process, whereas the significance does not survive.

As already stated, the AQI bits are, for mathematical reasons alone, the simplest possible structures within quantum physics. Disregarding such insights into the quantum theoretical contexts, one will obviously be inclined to search the simple in the spatially small, in line with the millenia old tradition.

- The discussion about the fundamentals of the description of nature is aggravated by the ancient prejudice that simplicity is to be found in spatial smallness.

And yet, the quantum theory has demonstrated since more than 100 years that this is an error.[13] If one tries to approach the problem of interactions with the prejudice of smallest elementary components, there will be no solution. Things look more promising using the protyposis concept, which leads to the actually simplest structures, and which, by the way, has also allowed us to explain, in an almost trivial way, the information behaviour at and in black holes.[14]

The various attempts to unify the three (non-gravitational) interactions within a single one, by introducing increasingly larger groups (which contain the mentioned three local gauge groups as sub-groups), do not at all lead to simple structures. Instead, an inflation of increasingly complex particle constructs can be witnessed, where the exemplars – in case they can be generated experimentally at all – prove to be ever less durable, ever more energy-rich and more complicated. We definitely do not deny the possibility to generate systems experimentally of such a complexity as predicted by those theories. However, the case of actually simple and thus fundamental structures is a different matter.

For the derivation of the interaction structures, in which the force quanta can appear as real particles in space and time, i.e., for the weak and the electromagnetic interaction with the gauge groups SU(2) and U(1), respectively, a recourse to the complex number structure at the core of quantum theory was not needed.[15]

However, that structure is essential if all three gauge groups are to be treated. Then it has to be considered that the quantum-theoretical description of nature amounts to a kind of “double-entry bookkeeping”, accounting for both the ‘facts’ and the ‘possibilities’. Crucially to this end is the use the complex numbers, which, in a way, may be interpreted as a duplication of the real numbers.

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[10] Görnitz, Th. (2011b)
[11] Görnitz, Th. (2014)
[12] See e.g. Preskill J.(1992); Bradler, K., Adami, Ch. (2014); Modak, S. K., Ortíz, L., Peña, I., Sudarsky, D. (2015); Hawking, St. W., Perry, M. J., Strominger, A. (2016)
[13] See e.g. Görnitz, Th., Görnitz, B. (2016), chapters 9.5, 9.10, 9.11, 10.3
[14] Görnitz, Th. (1988a), Abstr. QTh I, Görnitz, Th., Ruhnau, E,: (1989)
[15] Görnitz, Th. (2014).

4 Why is quantum theory defined on the field of the complex numbers?

In a talk at a conference dedicated to the 100th birthday of Erwin Schrödinger, Chen Ning Yang[16] quoted from a lecture on quantum mechanics given by Paul Dirac. The topic here is the non-commutability of operators, often presented as the essential feature of quantum theory in the literature. Dirac said

> "The question arises whether the noncommutation is really the main new idea of quantum mechanics. Previously I always thought it was but recently I have begun to doubt it and to think that maybe from the physical point of view, the noncommutation is not the only important idea and there is perhaps some deeper idea, some deeper change in our ordinary concepts which is brought about by quantum mechanics."

Dirac then continued, according to Yang, as follows:

> "So if one asks what is the main feature of quantum mechanics, I feel inclined now to say that it is not noncommutative algebra. It is the existence of probability amplitudes which underlie all atomic processes. Now a probability amplitude is related to experiment but only partially. The square of its modulus is something that we can observe. That is the probability which the experimental people get. But besides that there is a phase, a number of modulus unity which can modify without affecting the square of the modulus. And this phase is all important because it is the source of all interference phenomena but its physical significance is obscure."

How can the emergence of these complex numbers in physics be rationalized? Let us recall that classical physics is the physics of objects and facts. Its measured values are facts and thus represented by real numbers.[17]

However, in everyday life it is natural that not only facts but also those possibilities we expect cause effects. Quantum theory as the "physics of possibilities"[18] is to be understood as taking into account the finding that even in the inanimate nature future possibilities can create effects in the present.

Also within classical physics, possibilities are of course discussed. However, here possibilities are merely the consequence of insufficient knowledge of an 'observer' describing the system. Irrespective of whether they are known, the facts are entirely fixed and well defined according to the model of classical physics. For example, this applies to the statistical description in classical statistical thermodynamics.

It should be obvious though that the insufficient knowledge of an observer does not in any way affect the actual behaviour of the system.

This is in striking contrast to the possibilities in quantum theory, which can be designated as real or actual, because they can actually effectuate something. It is therefore important for their description that they are not represented on the same number axis as the real numbers labelling the facts.

Here further explanation is required. Possibilities generating actual effects refer necessarily to the future. That is, here the time evolution of the wave function is crucial, and the use of complex wave functions becomes mandatory. In the time-independent (static) mode, by contrast, wave functions can be and often are real functions, as already a cursory look at textbooks in quantum theory will show. For example, the energy eigenstates of the harmonic oscillator are real functions, as is the ground state of the hydrogen atom (the degenerate energy eigenstates can be chosen real as well, though then they are no longer eigenfunctions of the z-component of the angular momentum). In principle, any square integrable (and continuous) real function can be interpreted as a permissible wave function. The time-dependent Schrödinger equation (TDSE), governing the time-evolution of a wave function, explicitly introduces the imaginary unit *i*. As a consequence, even an initially real wave function becomes complex with the onset of the time evolution. Stationary states, that is, energy eigenstates are a special case, as here the time-dependence of the wave function consists in a trivial complex phase factor. A stationary state represents the presence of a fact, namely, the fact that the system assumes a precise energy value given by the respective energy eigenvalue. This fact does not change in the continuous time-evolution according to the TDSE.

- The possibilities that generate effects require something like a "second number axis" for the mathematical description.

However, a simple transition into a two-dimensional real description would not be adequate here, since in a real plane the two axes would be completely independent from each other.

For possibilities, however, it is crucial that some of them can become factual in the course of time. So for quantum theory the two envisaged axes must somehow be related to each other.

[16] Yang, C. N. (1987), we thank the Referee for this hint
[17] Görnitz, Th. (2011a).
[18] Görnitz, Th, Görnitz, B. (2002) Annex 14.2

Another aspect of quantum theory is its henadic, i.e. at unity aiming, structure. Here, a mathematical structure is needed that from the outset guarantees such a holistic behaviour.

- The solution for the structure required here had early been found in the re-interpretation of the two-dimensional real plane as a one-dimensional complex number ray; in addition it was supposed that the state functions, which constitute the behaviour of the quantum system, must, in principle, be complex differentiable.

The usual wave functions are complex-valued functions of real variables, such as the spatial coordinates or the momentum coordinates. The requirement of complex differentiability analytic behaviour ensures the analytic behaviour of these functions. This property is necessary to obtain a henadic structure. Note that also the scattering matrix is required to be analytic.

The basic importance of analyticity of the state functions has been recognized earlier. In 1959, Eugene Wigner gave his famous talk on "The Unreasonable Effectiveness of Mathematics in the Natural Sciences".

In striving for an answer to Wigner's assessment one has to consider the following: Mathematics can be understood as the science of possible structures, and physics as the science of the structures acting in nature. Structures in physics? emerge as a result of deeming unessential and ignoring certain aspects of the particular phenomena in the particular situations. Accordingly, it is hardly surprising that the structures dealt with in physics, if actually understood, can be treated mathematically.

In the mentioned talk Wigner characterizes the quantum-physical contexts as follows: [19]

> Let us not forget that the Hilbert space of quantum mechanics is the complex Hilbert space, with a Hermitean scalar product. Surely to the unpreoccupied mind, complex numbers are far from natural or simple and they cannot be suggested by physical observations. Furthermore, the use of complex numbers is in this case not a calculational trick of applied mathematics but comes close to being a necessity in the formulation of the laws of quantum mechanics. Finally, it now begins to appear that not only numbers but so-called analytic functions are destined to play a decisive role in the formulation of quantum theory.

The requirement of analyticity is important for the understanding the fundamental features. Structures deriving thereof appear, for example, in the so-called second quantization, briefly addressed below. For practical purposes, though, that requirement needs not rigorously be maintained. Often it is useful to simplify problems by applying suitable limits. Such mathematical simplifications, possibly even allowing for exact solutions, may, among others, serve pedagogical purposes as to better illustrate essential physical structures and the behaviour of quantum systems. A well-known textbook example is the particle moving in a rectangular potential well. While this model allows one to understand important physical aspects, such as the behaviour of energy eigenvalues for confined particles, other features get lost as a consequence of the limits supposed in the model. In nature, there are no such things as sharp edges or infinitely high potential walls, however easily and sensibly they may be postulated in mathematical contexts. Obviously, such „unnatural" postulates can and will be at odds with the requirement of analyticity. .

Often it proves useful to allow coordinates to become complex. In scattering theory virtual particles are referred to as off-mass-shell. Here E^2 or p^2 become negative, which means imaginary energy and momentum values.

Moreover, imaginary coordinates allow one to change between a quantum and classical description. According to the paper „Complex Coordinates and Quantum Mechanics" by F. Strocchi:

> „For example, the correspondence between Poisson brackets and commutators is not arbitrary: the Poisson bracket of two "classical" phase functions is in fact the mean value of the commutator between the corresponding operators." [20]

This shows once again that quantum theory is the more accurate description of the reality, and classical physics providing an averaging thereof.

The use of complex coordinates has proven useful in the field of atomic and molecular physics as well. [21] In the treatment of metastable states, for example, the energy aquires an imaginary component. Due to the quantum theoretical equivalence of the spatial and momentum representations, analytical behaviour is supposed with regard to the spatial coordinates as well. [22]

The analytical structure is the mathematical equivalent to holism, being the distinctive feature of quantum theory.

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[19] Wigner, E. P. (1960), we thank the Referee for this hint
[20] Strocchi, F. (1966)
[21] See e.g. Rescigno, T. N., McCurdy, Jr. C. W. and Orel, A. E. (1978); Moiseyev, N. (1998);
[22] Reinhard, W. P. (1982)

Very generally speaking, a holism is also found in the so-called Lie groups. These groups are characterized by the fact that a small neighbourhood of the identity element is sufficient to derive from it the whole group structure. In this sense, Lie groups can be seen as a generalization of analytic functions.

An advantage of the analytical functions is that they allow for a representation of stationary states such that a complex phase is changing while an associated expectation value remains constant. Then possibilities may change for a system, while the factually recognizable state might appear as unchanged. In other words, possibilities can be in stationary modes. Such conditions could be designated as virtual changes that factually appear as static.

Since quantum theory is certainly a genuine scientific theory, i.e., it represents a deterministic structure, the description of the quantum systems has to be made by using functions that allow for such deterministic structure by satisfying a differential equation with regard to space and time coordinates.

Here it has to be recalled that in quantum theory the deterministic development refers to possibilities, but not facts, which may become real within the scope compatible with the possibilities.

- Probability calculation is carried out in quantum theory by exceeding the field of real numbers and working likewise with complex numbers.

This expansion of the numbers is obtained by adding the imaginary unit ›i‹, the square root of ›-1‹. In this way, quantum theory takes into account that these real possibilities achieve effects and may influence each other, since they are objective and not merely reflecting the lack of knowledge of an observer. For example, an interaction of possibilities may make otherwise possible facts impossible.

As the well-known double-slit experiment shows, a photon or an electron behaves differently depending on whether it can move controlled or uncontrolled through the slits. When both slits are open and no control of the passage is in place, then there is the effect that certain spots on a detection screen will not be reached, although the very same spots are accessed by the particles if only one of the slits (irrespective which) is open.

For a description in terms of a differential equation for complex functions, the functions must be complex differentiable. This means they have to comply with the Cauchy-Riemann differential equations:

Be $f(z)$ a function of the complex variables $z = x + iy$,

$$f(z) = f(x,y) = f(x + iy) = u(x,y) + iv(x,y) \tag{1}$$

It is then necessary for the differentiability of $f(z)$ that the following applies for the two real functions u and v:

It is generally known that a complex differentiable function can be differentiated as often as desired. It can thus

$$\begin{aligned} \frac{\partial u}{\partial x}(x,y) &= \frac{\partial v}{\partial y}(x,y) \\ \frac{\partial u}{\partial y}(x,y) &= -\frac{\partial v}{\partial x}(x,y) \end{aligned} \tag{2}$$

be developed into a power series and is then designated as analytical function:

$$f(z) = \sum_{n=0} a_n z^n \tag{3}$$

- This *series expansion* is not only an interesting mathematical structure; it also has a physical equivalence in the form of the so-called *second quantization*.

The essential feature of an analytical function is that any piece of it defines the entire function. Therefore in mathematics the analytical functions represent the holism that distinguishes quantum theory from classical physics

The structure of the so-called “second quantization”, which however represents the basic structure of quantum theory at all[23], becomes visible in the clearest way, when the Fock space representation is used. Here the state space $\mathcal{H}$ of a quantum field is represented as an infinite sum over the state spaces $\mathcal{H}_n$ of n quantum particles.

$$\mathcal{H} = \bigoplus_{n=0}^{\infty} \mathcal{H}_n \tag{4}$$

[23] Görnitz, Th.(1999)

The n-particle state space H_n is the tensor product of *n* one-particle state spaces H_1

Depending on whether it is a Bose or a Fermi field, the state spaces H_n are still to be symmetrized or anti-symmetrized.

$$H_n = \bigotimes_1^n H_1 = H_1 \otimes H_1 \otimes \ldots \otimes H_1 \qquad (5)$$

The changes of the states of a quantum field are obtained by generation or annihilation of those particles that were introduced as field quanta.

This series structure represents a central aspect of quantum theory. Essentially in such a way a quantum particle can be constructed from quantum bits, the actually simplest quantum structures.(see also appendix 1)[24]

5 Interaction implies a division into separate spaces

- To introduce interaction it is furthermore necessary to 'break up' the henadic (= aiming for unity) structure of quantum theory, since ultimately the term 'interaction' is useful only for things separated from each other.[25]

Interaction implies separated objects. Accordingly, it is the natural structure of classical physics, representing reality as an accumulation of separated objects.

Since the beginning of theoretical mechanics it has been known that for the description of the interaction of two point masses, each mass requires its own coordinate space.

The simple case of two interacting particles in space is usually treated by re-writing the problem in terms of center-of-gravity and relative coordinates. Here it is easily overlooked that each particle has a space of spatial and momentum coordinates of its own. According to our view and in nature both particles are placed at different locations in the same space, while in the mathematical description each occupies its own 'cosmos' of coordinates. The action of force due the other particle arises, so to say, from another cosmos. Therefore, it is necessary for the description of the interaction of a particle with an outer force to provide – in addition to the Minkowski space – another space from which the description of force action can be constructed.

In the following it will be shown how the interaction terms of the three gauge groups are created from this separation and, in addition, from the involvement of the possibilities, for which quantum theory shows that, in addition to facts, they can generate effects as well.

It has already been explained that the mathematical structure of quantum theory is incompatible with a concept of interaction.[26] Even if a description was started with two separate objects, the tensor product structure of quantum theory would ensure that a new unity would be created – and the idea of interaction makes no sense for a unity. This means that a separation into two objects is mandatory in the sense of the dynamic layering structure,[27] and the tensor product structure of quantum theory must not be fully applied. Since the very beginnings of quantum theory Bohr has been insisting that classical physics is indispensable in order to speak about quantum theoretical results. Moreover, a precise analysis shows that a strict concept of interaction can only be formulated within the mathematical framework of classical physics. On the other hand, it is evident that the existence of all the objects described by classical physics can only be explained by quantum theory. This means that for a good description of nature we need both parts of physics, classical physics and quantum physics. This mutual relationship is referred to as "dynamic layering structure" or, referring to the dynamics in these phenomena, as "dynamic layering process".

Of course, while insisting in the quantum theoretical unity, one should not throw the baby out with the bath water. For example, in the treatment of the proton-electron interaction in a hydrogen atom one normally resorts to the classical Coulomb potential, governing the motion of the electron. Obviously, possible tensor structures are entirely irrelevant here. This applies to atomic and molecular structures, in general, where for good reasons the pertinent approximation procedures avoid an exaggerated „unity picture". In all electromagnetic interactions, such as encountered in chemistry, the interaction strengths are much too small as to envisage the generation of massive particles; and also effects of quantum field theory are so small that they safely can be neglected in most cases.

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[24] Görnitz, Th., Graudenz, D., Weizsäcker, C. F. v. (1992), Görnitz, Th., Schomäcker, U. (2012)
[25] Görnitz, Th. (2014)
[26] Görnitz, Th. (2014)
[27] Görnitz, Th. (1999) Kap. 5; Görnitz, Th., Görnitz, B. (2002) Appendix 14.4; Görnitz, Th. (2010)

In a very accurate description of the hydrogen atom, though, it becomes apparent that it is not simply a „two-particle system". Then the Coulomb field of nucleus and electron proves to be an ensemble of virtual photons and virtual electron-positron pairs. For this system, now to be described as a whole, the notion of interaction becomes problematic.

6 Actually elementary structures establish the structure of the space-time

Carl Friedrich v. Weizsäcker was the first to postulate that the three dimensions of the physical or position space are a necessary consequence of quantum theory. [28]

Nowadays, this understanding can be formulated as follows: All states of a quantum bit of the protyposis span an irreducible representation of the two groups U(1) and SU(2).

It follows from the theory of the compact groups that all irreducible representations of such a group can be realized in subspaces of that Hilbert space generated by the square-integrable functions on the parameter space of this group. For this purpose, it is necessary to consider this group as its own maximal homogeneous space.[29]

The SU(2) group manifold is the three-dimensional surface of a four-dimensional sphere. Accordingly, like in Weizsäcker's Ur-theory, the protyposis concept entails that the SU(2) group establishes the mathematical description of the cosmic position space.

This assumption is supported by the group-theoretical feature implying that everything that can be represented by quantum bits, i.e. all quantum particles and quantum fields, can be represented by functions on this maximal homogeneous space of the group SU(2).

From three physically plausible assumptions a cosmology follows that matches the observational data well and from which the validity of Einstein's equations can be concluded.[30] For this cosmology, it is sufficient to demand the following:

- The Planck relation of an inverse proportionality between characteristic length and energy is generally applicable. (With increasing energy, the [Compton] wavelength decreases.)
- There is a distinguished speed (which is usually designated as vacuum speed of light *c*).
- The first law of thermodynamics is valid ($dU + pdV = 0$).

From these requirements, the definition of a universal cosmic time results, and, further on, the Robertson-Walker metrics of a closed cosmos that is expanding with speed of light referred to this time. [31]

7 Introduction of interaction

Interaction is a conception valid for distinct objects in space, i.e., particles or fields. Quantum field theory shows that quantum fields can be understood as assemblages of quantum particles. Thus the essence of interaction can be understood if the interaction of particles has been explained.

As was established some time ago, there is a way in which quantum particles can be constructed from quantum bits. A short exposition is given in the appendix 1.

We have already discussed the specific status of gravitation, which is expediently not to be formulated as gauge theory. In the following, the three other types of interaction shall be treated.

To get access to these types of interaction, we have started from a quantum particle in the Minkowski space.[32] The quantum particles shall be defined as elementary objects. This means that no possible internal structures need to be taken into account. Their states can be characterized according to irreducible representations of the Poincaré group.

Normally, the starting point for the description of interaction is the interaction-free movement. In case of a free particle, the possible translations are generated by the momenta. In the quantum-theoretical description, their generators are the derivatives referred to the space-time coordinates of the Minkowski space.

$$P_k = -i\, \partial/\partial x_k \qquad (6)$$

[28] Weizsäcker, C F v, (1955, 1958, 1985, 2006)
[29] Görnitz, Th. (1988a), (1988b)
[30] Görnitz, Th. (2011b)
[31] Görnitz, Th. (2011b)
[32] Görnitz, Th. (2014)

Now one can reflect about how the concrete form of the momentum changes, when there are forces.

7.1 Electromagnetic and weak interaction

The treatment of electromagnetic, weak, and strong interaction in terms of local gauge interactions with the gauge groups U(1), SU(2), and SU(3), respectively, has proven empirically well founded for a long time. At the core of this description is the replacement of the usual derivative by a covariant derivative,

$$\partial/\partial x_k \quad \rightarrow \quad \partial/\partial x_k + A_a{}^k T^a \tag{7}$$

Here the T^a denotes the generators of the Lie algebra of the respective gauge groups. As mentioned in Chapter 1, so far there is the issue why exactly these three gauge groups are required and how the transition to the covariant derivatives can be rationalized. Another issue is the fact that the gauge bosons for U(1) and SU(2) can appear as real objects in the vacuum, whereas the gauge bosons of SU(3) have to be described as virtual particles, that is, structure quanta.

As explained in Chapter 5 and discussed, at greater length, in a previous paper[33], in establishing interaction the AQI bits forming the particles are associated with another ‚cosmos of description‘ than the AQI bits generating the interaction.

Addressing this point, we have demonstrated that the U(1) and SU(2) group generators associated with displacements in the maximal homogenous space of these groups, i.e. the group manifold itself, must augment the momentum generators of the Minkowski space.

The parameter space of the U(1) group is one-dimensional, that of the SU(2) group three-dimensional. The related generators of the Lie algebras shall be designated using ι and τ^a. The generators τ^a of the SU(2) can be represented by the well-known Pauli-matrices. (see e.g. appendix 2)

As is generally known, a group element in the neighbourhood of the group unit can be approximated for the U(1) group by

$$g = exp\{i\, A\, \iota\, \} \approx 1 + i\, A\, \iota \tag{8}$$

and for the SU(2) group by

$$g = exp\{i\Sigma\, B_a \tau^a\, \} \approx 1 + i\, \Sigma\, B_a \tau^a \tag{9}$$

if the series expansion of the exponential function is truncated after the first power.

- From the coupling of the momentum in the Minkowski space, i.e. the translations that lead to a change of location there, with the “translations” in the geometry of the interaction partner, a substitution results for the momentum operator with the following form:

$$P^k \rightarrow -i\partial/\partial x_k + g_1 A^k\, \iota + g_2 B^k{}_a \tau^a \tag{10}$$

with g_1 and g_2 being two coupling constants, which cannot be further specified from the considerations made so far.

As has been known for a long time, the descriptions of the weak and of the electromagnetic interaction in the Minkowski space lead to this structure.

- The protyposis concept of fundamental and simplest quantum structures affords an explanation of exactly this form of the interaction.

However, in the case of strong interaction, these considerations are not yet sufficient.

7.2 Strong interaction

- The strong interaction differs from the weak and the electromagnetic interaction in that here all ideas of interacting *particles* collapse.

The only thing known for sure about quarks and gluons is that they do not exist, at least not as free objects in space and time.

- Of course, quarks and gluons exist as incorporated structures, and without them the internal structure and the interaction of the hadrons would remain incomprehensible.

The attempt to isolate a quark leads to the formation of a quark-antiquark pair at the ‘breakpoint’. This means when trying to release a quark, always a meson will be produced, i.e. a quark-antiquark structure. It remains to be established that quarks and gluons have only virtual existence.

[33] Görnitz, Th. (2014)

- However, one of the fundamentals of quantum theory being that not only facts but also possibilities can create real effects, it must be anticipated that the quark-gluon structures are very significant in the description of the physical processes.

Of course, they have to be understood as genuine quantum phenomena, reflecting the possibility characteristics of quantum theory.

In contrast to the weak and electromagnetic interaction, where forces are transmitted by real quanta in space and time, we here have to rely exclusively on virtual quanta.

In analogy to the extension of real to complex numbers in the full quantum description, this suggests to postulate a duplication of the structures associated with the interactions based on real quanta. In the "real quanta interactions" we have an SU(2)×U(1) structure, which then would have to be duplicated. In appendix 2 it is shown that the four-dimensional group SU(2)×U(1) is a subgroup of the group SU(3).[34]

So the question arises how the transition from the SU(2)×U(1) structure to a SU(2)×U(1) plus SU(2)×U(1) structure is to be interpreted. The answer is simple: We are led exactly to the SU(3) structure.

The SU(3) group is an eight-dimensional compact group, and a parameterization of SU(3) can be obtained which matches the duplicated structure of the SU(2)×U(1) groups.

Such a structure is called the Cartan decomposition.[35] For a group *G* with the Lie algebra ***G*** and a subgroup K with Lie algebra ***K***, the coset being ***P***, there is the Cartan decomposition of the Lie algebra ***G*** if

$$\boldsymbol{G} = \boldsymbol{K} + \boldsymbol{P} \tag{11}$$

with

$$k_i \in \boldsymbol{K} \quad \text{and} \quad p_i \in \boldsymbol{P} \tag{12}$$

and if the corresponding structures hold for the elements

$$\left[k_i, k_j\right] \in \boldsymbol{K}; \quad \left[p_i, p_j\right] \in \boldsymbol{K}; \quad \left[k_i, p_j\right] \in \boldsymbol{P}; \tag{13}$$

In case ***K*** and ***P*** have the same dimension, ***P*** can, as manifold, be a copy of ***K***. In this case the structure (13) is the same as in the relations (14) for real and imaginary numbers:

$$\textit{real} \times \textit{real} = \textit{real}\ ,\ \ \textit{imaginary} \times \textit{imaginary} = \textit{real}\ \text{ and } \textit{real} \times \textit{imaginary} = \textit{imaginary} \tag{14}$$

The eight parameters of the SU(3) shall be designated by the vector (α,β,γ,θ,a,b,c,φ). We follow Byrd's representation: [36]

Dropping the redundancies, we arrive at the following product representation,

$$D(\alpha,\beta,\gamma,\theta,a,b,c,\varphi) = e^{(i\lambda 3\alpha)} e^{(i\lambda 2\beta)} e^{(i\lambda 3\gamma)} e^{(i\lambda 5\theta)} e^{(i\lambda 3a)} e^{(i\lambda 2b)} e^{(i\lambda 3c)} e^{(i\lambda 8\varphi)} \tag{15}$$

for an arbitrary element D of SU(3). This can be written as

$$D(\alpha,\beta,\gamma,\theta,a,b,c,\varphi) = D^{(2)}(\alpha,\beta,\gamma) e^{(i\lambda 5\theta)} D^{(2)}(a,b,c)\, e^{(i\lambda 8\varphi)} \tag{16}$$

where again the D denotes an arbitrary element of SU(3), and $D^{(2)}$ is an arbitrary element of SU(2) as a subset of SU(3). The λ_i are 3×3-matrices, therefore also D and $D^{(2)}$ are 3×3-matrices (see appendix 2). The elements with λ_1, λ_2, λ_3, and λ_8 belong to the subgroup *K* and from it with λ_5 the elements of the coset *P* can be created. The two exponential functions $e^{(i\lambda 8\varphi)}$ and $e^{(i\lambda 5\theta)}$describe the two U(1) manifolds as subgroup and coset in SU(3), respectively.

The parametrization chosen by Byrd is referred to as „non-canonical“. The canonical parametrization uses the exponential mapping of the Lie algebra onto the group as indicated in Eqs. (8) and (9). In the latter case, there is a convenient relation for the parameters in a one-parameter subgroup:

$$D(\alpha)\, D(\beta) = D(\alpha + \beta) \tag{17}$$

With regard to the different parametrizations, one may refer, for example, to the following hints by Gilmore[37]:

[34] Böhm, M. (2011), p. 231
[35] see e.g. Böhm, M. (2011), p. 304 ff; Knapp, A. W. (2002)
[36] Byrd, M.: (1998)
[37] Gilmore (2005), p. 149

“This parameterization [the canonical] is obtained by the EXPonential mapping of the Lie algebra onto the Lie group. In this parameterization, every straight line through the origin of the algebra exponentiates onto a one-dimensional abelian subgroup.

To obviate the impression that noncanonical parameterizations are anathema, we propose now to deal with a number of them. This is not an empty academic exercise: we have a number of motivations for such a discussion.

1. Mathematical reasons. For one thing, it is often difficult to construct the canonical parameterization of a classical Lie group using the EXPonential mapping. It is even more difficult to construct canonical matrix representations of a group by the canonical mapping of the algebra's representations onto the group's representations with the EXPonential mapping.

2. Physical reasons. We will eventually want to associate physical opera-tors with elements in Lie algebras and groups. For instance, it is often useful to associate shift-up and shift-down operators like J+ and J_ with operators in a Hamiltonian which cause transitions to higher and lower energy levels. Then it becomes necessary to compute matrix elements of ordered operator products within particular representations. The existence of noncanonical parameterizations allows the construction of generating functions for products of operators in normal and symmetrized orderings.”

In a particular case, one will chose the parametrization that is best adapted to the situation. Often this will be a canonical parametrization. The advantage of Byrd’s non-canonical parametrization is that it makes manifest the SU(3) product structure according to Eq. (16), being central for our considerations, and, moreover, the analogy to the formula (14).

Since SU(3) is semi-simple, it has only one connected component. Thus the given parameterization in (16) covers the whole group.

- Formula (16) is crucial for our considerations. It indicates precisely how the protyposis concept leads us to the SU(3) structure of the strong interaction.

To establish the ‚acting possibilities‘ in quantum theory a duplication of the manifold of real numbers had to be introduced together with the Cartan-type product structure (14). This entailed the complex numbers. In an entirely analogous way, the space of the protyposis AQI bits associated with the interaction, that is the SU(2)×U(1) manifold, is to be duplicated and endowed with the Cartan structure (13). As a result, one obtains an additional interaction, which, being quantal, acts only via virtual or structure quanta, and is represented by the SU(3) gauge group.

The formula (10) for the electromagnetic and weak interaction is to be supplemented with the generators λ^a of SU(3) yielding

$$P^k \rightarrow -i\partial/\partial x_k + g_1 A^k \iota + g_2 B^k{}_a \tau^a + g_3 C^k{}_a \lambda^a \qquad (18)$$

It should be emphasized once more that the SU(3) structure can be explained as resulting from a quantum-theoretically motivated duplication, analogous to the transition from real to complex, which applies to the case in which the interaction is mediated by virtual rather than real quanta. Accordingly, quarks and gluons cannot appear as free particles in vacuum, just like phonons cannot appear outside a solid. Nonetheless, all such quantum structures produce real effects.

Appendix 1 Quantum particles from the quantum bits of the protyposis

- A quantum particle is defined by a fixed mass and a certain spin, and by the fact that all its states span an irreducible representation of the Poincaré group.

First it is necessary for this mathematical structure to define operators for the generation and annihilation of the quantum bits. One of the symmetries at these quantum bits is the complex conjugation, which effects a so-called anti-linear representation. So it could be seen very early[38] that it is useful to allow linear representation by duplication of the state space. Weizsäcker spoke about urs and anti-urs in this connection.

The operator generating a quantum bit shall be $a_s^\dagger$, and the operator annihilating a quantum bit shall be a_s .

To construct also massive particles, it is necessary to introduce parabose commutation rules for the generation and annihilation operators

[38] See e.g.: Weizsäcker (1985) p. 406

$$\tfrac{1}{2}[\{a_r, a_s^\dagger\}, a_t] = -\delta_{st} a_r$$
$$[\{a_r, a_s\}, a_t] = [\{a_r^\dagger, a_s^\dagger\}, a_t^\dagger] = 0 \qquad (19)$$

The state index *s* or *r* or *t* can range from 1 to 4.

The effect on the vacuum of the protyposis then results as:

$$a_s a_r^\dagger |\Omega\rangle = \delta_{rs} p |\Omega\rangle \qquad (20)$$

with |Ω> being the vacuum of protyposis, *p* the parabose order, *p=1* the Bose statistics.

The vacuum in the Minkowski space, the Lorenz vacuum |0〉**,** is an eigenstate of the Poincaré group with zero mass, energy and spin. While all other irreducible representations of the Poincaré group span an infinite-dimensional state space, the Lorenz vacuum |0〉 corresponds to a representation with a one-dimensional state space.

- This vacuum can be characterized by the finding that no particle is to be found at each of the infinitely many points in the Minkowski space.

The particle vacuum in the Minkowski space shall serve as simple example for the relation of quantization to the analytical functions. This vacuum is – like a "normal" particle state – also an eigenstate of the Poincaré group. It proves to be an infinite sum of states of quantum bits, which are generated from |Ω>, the vacuum of the quantum bits of protyposis.

$$|0\rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{(-1)^{n_1+n_2}}{n_1! \cdot n_2!} \left(\frac{a_1^\dagger a_3^\dagger + a_3^\dagger a_1^\dagger}{2} \right)^{n_1} \left(\frac{a_2^\dagger a_4^\dagger + a_4^\dagger a_2^\dagger}{2} \right)^{n_2} |\Omega\rangle \qquad (21)$$

For the Lorenz vacuum, the annihilation of a quantum bit corresponds to the generation of the related anti-qubit.

$$a_1|0\rangle = -a_3^\dagger|0\rangle \qquad a_2|0\rangle = -a_4^\dagger|0\rangle \qquad a_3|0\rangle = -a_1^\dagger|0\rangle \qquad a_4|0\rangle = -a_2^\dagger|0\rangle \qquad (22)$$

In the same way, but much more complicated than the vacuum of the particles, the state of a quantum particle with rest mass can be represented as a Fock representation of states of quantum bits. An example[39] shall be the state of rest (momentum = 0) of a fermion with mass m and spin ½ in z–direction.
Due to increasing complexity, processing by means of a computer and with an adapted notation seems to be appropriate. For this purpose, notation has been adapted to an application in mathematica®[40] (with "Erzeuger" = generator, "Vernichter" = annihilator).

$$\begin{array}{ll} a_r^\dagger \Rightarrow e[r] & \text{(Erzeuger)} \\ a_r \Rightarrow v[r] & \text{(Vernichter)} \\ \{a_r^\dagger, a_s^\dagger\} \Rightarrow 2\,f[r,s] & \{a_r, a_s\} \Rightarrow 2w[r,s] \\ \{a_r^\dagger, a_s\} \Rightarrow 2d[r,s] & |0\rangle \Rightarrow lvac \end{array} \qquad (23)$$

With this computer-adapted notation, the ten generators of the Poincaré group are given the following form:

Translations:
P1 = (-w[2,3]-f[3,2]-w[1,4]-f[4,1] -d[1,2]-d[2,1]-d[4,3]-d[3,4])/2
P2 = I*(-w[2,3] +f[3,2] +w[1,4]-f[4,1] -d[1,2] +d[2,1]-d[4,3] +d[3,4])/2
P3 = (-w[1,3]-f[3,1] +w[2,4] +f[4,2] -d[1,1] +d[2,2]-d[3,3] +d[4,4])/2
P0 = (-w[1,3]-f[3,1]-w[2,4]-f[4,2] -d[1,1]-d[2,2]-d[3,3]-d[4,4])/2

[39] Görnitz, Th., Schomäcker, U. (2012)
[40] Görnitz, Th., Schomäcker, U. (2012)

Boosts:
M10 = I*(w[1,4]-f[4,1] +w[2,3]-f[3,2])/2 (24)
M20 = (w[1,4] +f[4,1]-w[2,3]-f[3,2])/2
M30 = I*(w[1,3]-f[3,1]-w[2,4] +f[4,2])/2

Rotations:
M32 = (d[2,1] +d[1,2]-d[3,4]-d[4,3])/2
M21 = (d[1,1]-d[2,2]-d[3,3] +d[4,4])/2
M31 = I*(d[2,1]-d[1,2]-d[3,4] +d[4,3])/2

The state of this massive particle turns out as infinite sum over differently weighed states of quantum bits. In this specific case, they comply with parabose symmetry. The parabose order p[0] is greater than 1. (Only massless objects can be generated with Bose symmetry). The symbols p[i] designate powers of the individual operators. The symbol * is used to identify the commutative product of numbers, and ** is used to identify a non-commutative product of the generation and annihilation operators. As an example we give the state of a massive fermion with mass *m* at rest. The momentum at rest is P0=m, P1=P2=P3=0, and the spin in z-direction is $s_z=1/2$. The expression has two parts:

$$\sum_{p[3]=0}^{\infty}\sum_{p[2]=0}^{\infty}\sum_{p[1]=0}^{\infty}\frac{(-1)^{(p[1]+p[2]+p[3])}\ (m)^{(2\,p[1]+p[2]+p[3])}}{p[1]!\,p[2]!\,p[3]!\,(2\,p[1]+p[2]+p[3]+p[0])!}*$$
$$\frac{(p[1]+p[2]+p[3]+p[0]-1)!}{(p[1]+p[3]+p[0]-2)!\,(p[1]+p[2]+p[0]-1)!}*$$
$$e[1]**f[4,2,p[3]]**f[4,1,p[1]]**f[3,2,p[1]]**f[3,1,p[2]]**lvac \tag{25}$$

$$+\sum_{p[3]=0}^{\infty}\sum_{p[2]=0}^{\infty}\sum_{p[1]=0}^{\infty}\frac{(-1)^{(p[1]+p[2]+p[3])}\ (m)^{(1+2\,p[1]+p[2]+p[3])}}{p[1]!\,p[2]!\,p[3]!\,(2\,p[1]+p[2]+p[3]+p[0]+1)!}*$$
$$\frac{(p[1]+p[2]+p[3]+p[0]-1)!}{(p[1]+p[3]+p[0]-1)!\,(p[1]+p[2]+p[0]-1)!}*$$
$$e[2]**f[4,2,p[3]]**f[4,1,p[1]+1]**f[3,2,p[1]]**f[3,1,p[2]]**lvac$$

Appendix 2: Remarks on the structure of the SU(3) group

In his paper "The Geometry of SU(3)"[41] Mark Byrd gives an overview on the geometry of the group manifold of SU(3).

With the "Euler angle" parameterization, presented by him, we can connect the structure deriving from the protyposis concept with this gauge group.

The Lie-algebra of the group is often represented by the 8 Gell-Mann matrices, named λ_i. They provide the most common representation in terms of 3 × 3 hermitian, traceless matrices.

$$\lambda_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\quad \lambda_2=\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix},\quad \lambda_3=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},$$
$$\lambda_4=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},\quad \lambda_5=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix},\quad \lambda_6=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}, \tag{26}$$
$$\lambda_7=\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix},\quad \lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}.$$

Abb. 1: The Gell-Mann matrices of the Lie algebra of SU(3)

[41] Byrd, M.: (1998)

The matrices λ_1, λ_2 and λ_3 are the Pauli matrices of SU(2), extended by a third line and column. The matrix λ_8, the generator of U(1) , commutes with λ_1, λ_2 and λ_3. Accordingly, SU(2)×U(1) is a subgroup of SU(3).

The whole set commutation relations can be listed in tabular form as follows:

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
λ_1	0	$2i\lambda_3$	$-2i\lambda_2$	$i\lambda_7$	$-i\lambda_6$	$i\lambda_5$	$-i\lambda_4$	0
λ_2	$-2i\lambda_3$	0	$2i\lambda_1$	$i\lambda_6$	$i\lambda_7$	$-i\lambda_4$	$-i\lambda_5$	0
λ_3	$2i\lambda_2$	$-2i\lambda_1$	0	$i\lambda_5$	$-i\lambda_4$	$-i\lambda_7$	$i\lambda_6$	0
λ_4	$-i\lambda_7$	$-i\lambda_6$	$-i\lambda_5$	0	$i\lambda_3 + i\sqrt{3}\lambda_8$	$i\lambda_2$	$i\lambda_1$	$-i\sqrt{3}\lambda_5$
λ_5	$i\lambda_6$	$-i\lambda_7$	$i\lambda_4$	$-i\lambda_3 - i\sqrt{3}\lambda_8$	0	$-i\lambda_1$	$-i\lambda_2$	$i\sqrt{3}\lambda_4$
λ_6	$-i\lambda_5$	$i\lambda_4$	$i\lambda_7$	$-i\lambda_2$	$i\lambda_1$	0	$-i\lambda_3 + i\sqrt{3}\lambda_8$	$-i\sqrt{3}\lambda_7$
λ_7	$i\lambda_4$	$i\lambda_5$	$-i\lambda_6$	$-i\lambda_1$	$i\lambda_2$	$i\lambda_3 - i\sqrt{3}\lambda_8$	0	$i\sqrt{3}\lambda_6$
λ_8	0	0	0	$i\sqrt{3}\lambda_5$	$-i\sqrt{3}\lambda_4$	$i\sqrt{3}\lambda_7$	$-i\sqrt{3}\lambda_6$	0

(27)

Abb. 2: The entries in the table are given by commuting the element in the first column with the element in the top row: [element from first column, element from top row] = element in table at the corresponding position.

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